

Free French Fries (Simulating Waiting Time)

A fast food restaurant has decided to give away a coupon good for one free order of French fries to one out of every 4 customers. The manager at the restaurant decided that it would randomly generate coupons with its food receipts so that it would not have customers waiting in line counting the number of people in front of them who received a coupon. Because the restaurant is using a random generating program in its registers, the probability of getting a coupon with every receipt is $1/4$. **On average, how many orders does one have to place before a customer gets 3 coupons for free French fries?**

This is an example of a waiting time problem. Statisticians often use simulations to model these problems – how long must one wait until the first or second or ... success occurs.

This situation is different from some of the problems you had before. In those problems you were counting the number of successes you had out of a fixed number. In this situation, you are counting how long it takes until you get a success. Because of this change in the problem situation, the simulation model will be slightly different from the simulations you have done in the past. A waiting time simulation is much more open-ended. You keep selecting until you have the required level of success. At that time, the trial ends and you start a new run of the simulation.

The 3 rules for simulations still apply.

Rule 1 – What are the assumptions that must be made in the problem?

In this case, we must assume that the probability that a free coupon appears with every food receipt $1/4^{\text{th}}$ of the time and that this probability remains the same for each receipt or order. It is also necessary to assume that getting a French fries coupon with a receipt is independent from receipt to receipt. This means that if you get a coupon on this order, it will not affect whether or not you get a coupon on the next order.

Rule 2 – How will I model a $1/4$ chance or a .25 probability that I get a free coupon with every order? In this problem, we are going to use a spinner to generate the probability of success.

Rule 3 – Perform many repetitions of each trial. A trial is a series of steps that stops when the situation of interest has been completed one time. The values that you get from many trials will be used to answer a question. In this case, one trial would be how many orders I would have to place until I get 3 coupons for free French fries.

- a) Draw a spinner to model a probability of $1/4$ and explain how you would use it.
- b) Explain what would constitute one trial or run. It is important to include not only what one success is, but that you keep spinning until you achieve 3 successes. Your explanation must include a stopping point.
- c) What are the numbers that you are going to record? You must keep an accurate record of your data and your results.
- d) Conduct 5 trials of your simulation and complete the table below.

Table of counts for coupons for free French fries	
Trial Number	Number of orders placed to get 3 coupons for French fries
1	
2	
3	
4	
5	
Total	

- e) On average, how many orders did you have to place until you got 3 free French fries coupons?
- f) What could you do so that you had a more accurate average or Expected Value for the number of orders you had to place?

- g) Pool the class's data and complete the chart below.

Table of counts for coupons for free French fries		
Number of orders to get 3 free French fries coupon	Frequency	Cumulative Frequency
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

(You may need to continue this chart for more values)

- h) Why does the first row of the above table start with a 3?
- i) What is the maximum number of orders one would have to place to get 3 free coupons?
- j) Why do you think this simulation is called a “waiting time” simulation?

- k) Based on the pooled data, what is the estimated probability that it will take exactly 8 orders to get 3 free coupons?

- l) Based on the pooled data, what is the estimated probability that it will take more than 6 orders to get 3 free coupons?

- m) Based on the pooled data, what is the average number of orders one would have to place before one got 3 free coupons?

- n) How does the answer in (m) compare to the answer you got in (e)? Which do you think is the more accurate of the two? Explain why.

SPINNER

